The Role of Algorithms in Teaching Problem Solving

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Recently, students in my course were assigned the following question (*1*).

> A sample of indium bromide weighing 0.100 g reacts with silver nitrate, $AgNO_3$, giving indium nitrate and 0.159 g of AgBr. Determine the empirical formula of indium bromide.

In virtually every recitation section, the students asked the TA to do this problem. Time and time again, the TA's told the students that the problem could be worked more or less like this

> Start by converting grams of AgBr into moles of AgBr. Convert moles of AgBr into moles of Br, and then convert moles of Br into grams of Br. Subtract grams of Br from grams of indium bromide to give grams of In. Convert grams of In into moles of In. Then divide moles of Br by moles of In to get the empirical formula of the compound.

During the next staff meeting, I asked my TA's to stop lying to the students. I suggested that (1) the technique for solving this problem that they had presented to their students had little to do with the process that they had used to solve the problem for the first time, (2) they had confused the process used to solve exercises with the process used to solve problems, and (3) the description given above was an algorithm for solving similar questions that they had constructed after they had solved this problem.

Definition of an Algorithm

Algorithms have been defined as "... rules for calculating something, especially by machine" (*2*). They are rules that can be followed more or less automatically by reasonably intelligent systems, such as a computer. A common algorithm among chemists is the factor–label approach to unit conversions, such as converting feet into inches or inches into centimeters. Other examples might include the fast Fourier transform, the road-map approach to stoichiometry questions, the rules some chemists use for automating the writing of Lewis structures, and the step-by-step process often used to predict the shape of a molecule from its Lewis structure.

The Difference between Exercises and Problems

Hayes's (*3*) definition of a problem reads,

> Whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap, you have a problem.

His definition provides a basis for distinguishing between two closely related concepts, exercises and problems. According to this definition, if you know what to do when you read a question, it is an exercise not a problem. The indium bromide question was a problem for every student in my general chemistry class, and for most of the teaching assistants as well. It was an exercise, however, for several of my colleagues who knew exactly what to do when they read the question.

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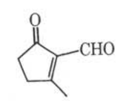
Status as a problem is not an innate characteristic of a question, it is a subtle interaction between the question and the individual trying to answer the question. It is a reflection of experience with that type of question more than intellectual ability. For example, I would expect that many chemists for whom the indium bromide question was an exercise would find the following question to be a problem, and vice versa.

> Design a synthesis for the following compound.



The Role of Algorithms in Working Exercises

Algorithms are useful for solving routine questions or exercises. In fact, the existence of an algorithm constructed from prior experience (*4*) may be what turns a question into an exercise. By listening to us in lecture, by reading examples in the text, and most importantly by working similar questions on their own, many if not most of our students construct algorithms that turn the following question into an exercise when it subsequently appears on an exam (*5*).

> What is the empirical formula of a compound of xenon and oxygen which is 67.2% Xe and 32.8% O?

Unfortunately, algorithms are not sufficient for answering exam questions that are more likely to be problems for our students, such as (*5*):

> 9.33 grams of copper metal was allowed to react with an excess of chlorine, and it was found that 14.6 grams of a compound of copper and chlorine were formed. What is the empirical formula of this compound?

It's not surprising, therefore, that most students in a survey of beliefs about chemistry agreed with the statement: Good teachers shouldn't ask students to figure out problems on an exam that they have not seen in class (*6*).

The Role of Algorithms in Working Problems

Johnstone (*7*) has suggested that a common source of difficulty in science is the overload that occurs when the demand on working memory exceeds its capacity. One solution to this overload is to help the student build strategies that decrease a task's demand on working memory. Johnstone described these strategies as tricks or techniques for simplifying problems and schemas for organizing prior knowledge. To some extent, these strategies are algorithms that automate individual steps in a problem.

Students who have not built algorithms for at least some of the steps in a problem, such as converting between grams and moles, will never solve the problem. There is more to working problems, however, than applying algorithms in the correct order. Before turning to the next section, try the following question (*1*).

A sample of a compound of xenon and fluorine was confined in a bulb with a pressure of 24 torr. Hydrogen was added to the bulb until the pressure was 96 torr. Passage of an electric spark through the mixture produced Xe and HF. After the HF was removed by reaction with solid KOH, the final pressure of xenon and unreacted hydrogen in the bulb was 48 torr. What is the empirical formula of the xenon fluoride in the original sample?

Now write down a summary of what you would tell your students if you decided to work this question in class.

Stages or Steps in Problem Solving

Forty years ago, Polya (8) proposed four stages in problem solving: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. While working the question given above, you undoubtedly spent a considerable amount of time on the stage Polya described as "understanding the problem". What is less certain is whether you went through a separate stage in which you devised a plan to solve this problem before carrying out the plan.

The steps that many people go through while working a "problem" such as this might be represented more or less as follows. You began by reading the problem, perhaps more than once. You then wrote down what you hoped was the key information, reread the question or a part of the question, drew a picture to help represent the question, and then tried something. Then you tried something else, and looked at where this led you. By gradually exploring or playing with the question you got closer and closer to the answer. It's possible that you never fully "understood" the question until you had an answer.

If you compare this reconstruction of the steps many of us take while solving a problem with the description we all too often give our students of how we solved the problem, you may understand why I asked my TA's to stop lying to the students, and you may also understand the role of algorithms in solving exercises versus problems.

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